TD 5

ex 1

Il s'agit d'une loi binomiale dont les paramètres sont $n = 5$ et $p = 0,51$

1) On cherche $P(X = 5)$

$$P(X = x) = C_n^x \, p^x (1-p)^{n-x}$$

$$P(X = 5) = C_5^5 \times 0,51^5 \times 0,49^0$$

$$= 0,51^5 = 0,0345$$

2) $$P(X = 2) = C_5^2 \times 0,49^2 \times 0,51^3$$

$$= \frac{5}{3!\,2!} \times 0,49^2 \times 0,51^3$$

$$= 0,3185$$

3) $$P(X \geq 2) = 1 - p(X < 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - (C_5^0 \times 0,49 \times 0,51^5 + C_5^1 \times 0,49^1 \times 0,51^4)$$

$$= 0,8$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Ex 2

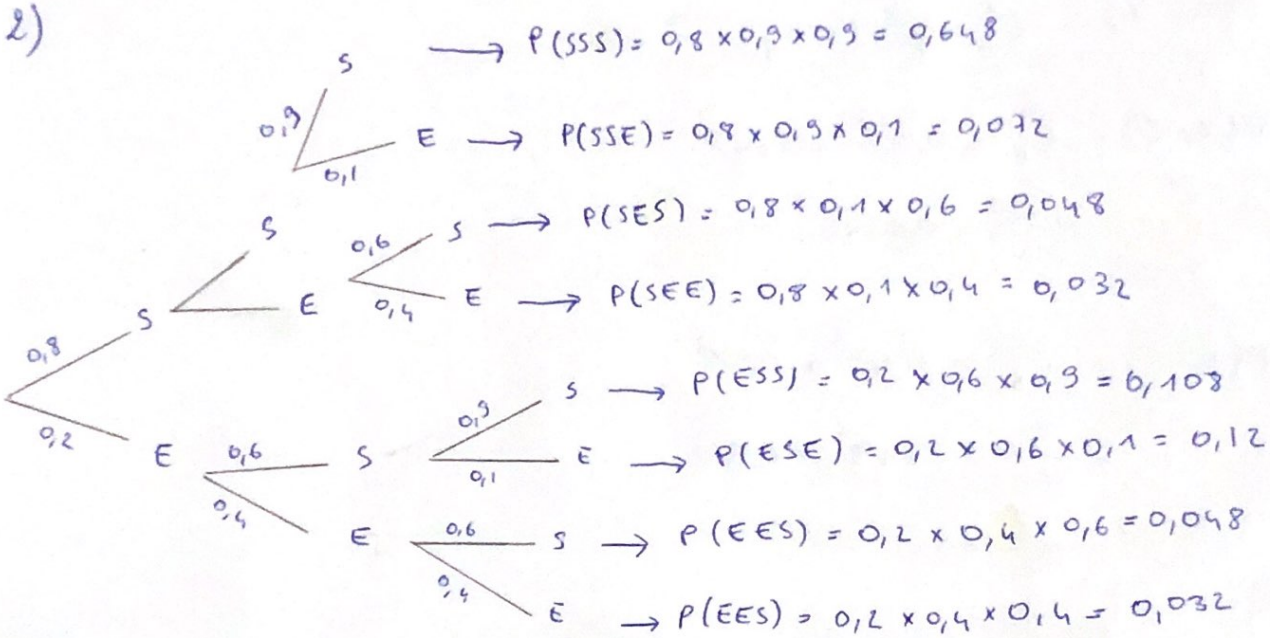
1) $X \rightsquigarrow \beta(3 ; 0,8)$

$P(X \geq 2) = P(X=2) + P(X=3)$

$= C_3^2 \times 0,8^2 \times 0,2^1 + C_3^3 \times 0,8^3 \times 0,2^0$

$= 0,896$

2)



3) $P(\text{au moins 2 ventes}) = P(SSS) + P(SSE) + P(SES) + P(ESS)$

$= 0,648 + 0,72 + 0,048 + 0,108$

$= 0,876$

Ex 3

1) loi de $X_n \rightsquigarrow \beta(6n ; p)$

$E(X_n) = 6n \times p$

$V(X_n) = 6n \times p \times q$

2) $u_n = P(X_n = n) = C_{6n}^n \, p^n \times q^{5n}$

$u_n = \dfrac{6n!}{n!\,5n!} \times p^n \times q^{5n}$

- Formule de Stirling : $\lim_{n \to +\infty} n! = \frac{n^n}{e^n} \sqrt{2\pi n}$

$\lim P(X_n = n) = \frac{(6n)^{6n}}{e^{6n}} \times \sqrt{2\pi \times 6n} \times$

$$\lim_{n \to +\infty} \left(P(X_n = n)\right) = \frac{(6n)^{6n}}{e^{6n}} \sqrt{2\pi \times 6n} \times \frac{e^n}{n^n \sqrt{2\pi n}} \times \frac{e^{5n}}{(5n)^5 \times \sqrt{2\pi 5n}} \times p^n \times q^{5n}$$

$$= \frac{(6n)^{6n}}{n^n \times (5n)^{5n}} \times \frac{\sqrt{2\pi}\, 6n}{\sqrt{2\pi n} \times \sqrt{2\pi 5n}} \times p^n \times q^{5n}$$

$$= \frac{6^{6n} \times n^{6n}}{5^{5n} \times n^{5n} \times n^n} \times \sqrt{\frac{3}{5\pi n}} \times p^n \times q^{5n}$$

$$= \left(\frac{6^6}{5^5} \times p \times q^5\right)^n \times \sqrt{\frac{3}{5\pi n}}$$